EDUCATIONAL MONOGRAPH

<u>An Example of Compensation Network Design</u>

prepared from

<u>Wide Band Phase Distortion Equalization</u>

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FOREWORD

This monograph was produced at Virginia Polytechnic Institute in a pilot program administered by Oklahoma State University under contract to the NASA Office of Technology Utilization. The program was organized to determine the feasibility of presenting the results of recent research in NASA Laboratories, and under NASA contract, in an educational format suitable as supplementary material in classwork at engineering colleges. The monograph may result from editing single technical reports or synthesizing several technical reports resulting from NASA's research efforts.

Following the preparation of the monographs, the program includes their evaluation as educational material in a number of universities throughout the country. The results of these individual evaluations in the classroom situation will be used to help determine if this procedure is a satisfactory way of speeding research results into engineering education.

ABSTRACT

This monograph presents an example of the design of a compensation network to compensate for time-delay distortion introduced by a nonlinear phase vs. frequency characteristic. The compensation network is a symmetrical lattice network and the design relations are developed by the use of image parameters.

INSTRUCTOR'S GUIDE FOR MONOGRAPHS

1. Educational level of the monograph - Senior level.
2. Prerequisite course material - The course material required for this monograph consists of the normal undergraduate network theory plus an introduction to the Image-Parameter Method.
3. Estimated lecture time required - One hour. This should be extended to two hours if it is necessary to include a brief introduction to the Image-Parameter Method.
4. Technical significance of the monograph - The material presented in this monograph illustrates a technique for compensation of time delay introduced by a nonlinear phase vs. frequency characteristic.
5. New concepts or unusual concepts illustrated - This monograph presents an example of lattice network design using the Image-Parameter Method.
6. How monograph can best be used - It is suggested that the material in this monograph be presented as lecture material illustrating the use of image parameters to design a lattice type compensation network. The problem may be used as either an example or as a home problem.
7. Other literature of interest to this monograph is given in the list of references.
8. Other reports reviewed by the editor in preparing this monograph - none.
9. Who to contact for further information - Technical Utilization Officer, NASA, Goddard Space Flight Center, Greenbelt, Maryland.
10. Note to Instructor: All uncolored pages of the instructors monograph are in the copies intended for student use.

AN EXAMPLE OF COMPENSATION NETWORK DESIGN

Introduction

A signal containing several frequency components can be distorted in many ways during passage through a system. This monograph considers one such type distortion and a method for the design of a network to compensate for this distortion. The distortion type considered herein could be referred to as time-delay distortion and arises in any system which has a nonlinear phase vs. frequency characteristic.

The Problem

The time delay of a system is given by

$$\tau_d = \frac{d\phi}{d\omega}$$

where ϕ and ω are the phase and frequency of the signal respectively. The ideal phase characteristic is linear and given by

$$\phi = k\omega + c$$

where k and c are constants. This ideal characteristic gives a constant time delay and all frequency components are transmitted with equal delay. Any variation from this ideal characteristic results in a frequency dependent time delay and a consequent distortion in output due to different delays for different frequency components.

In contrast to the ideal constant time-delay characteristic, some systems exhibit a time delay which is minimum over a band of frequencies and increases for frequencies outside this band. One such example of this type characteristics are those of the tape recorder typical of which are shown in Fig. 1.

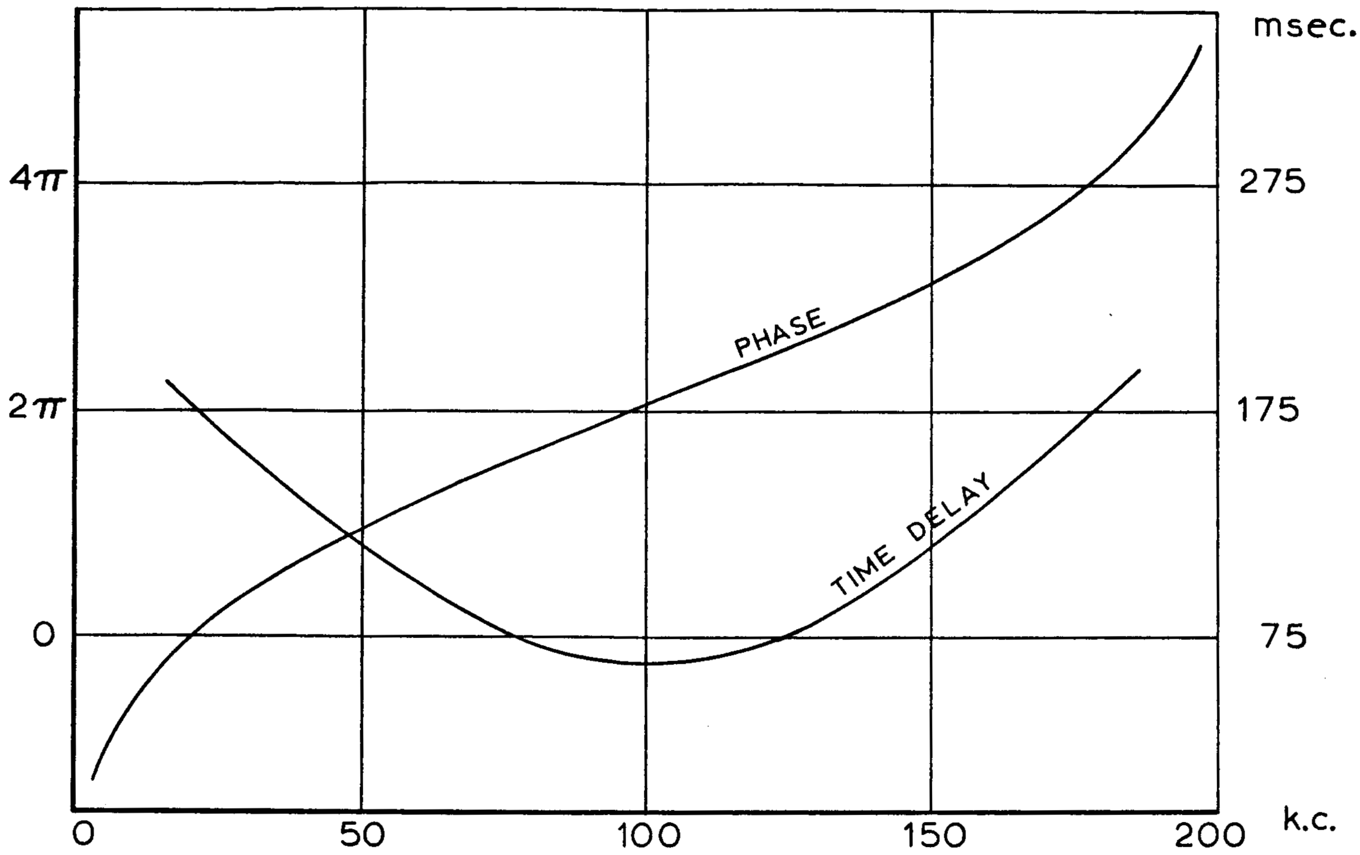


Fig. 1. A Typical Phase vs. Frequency Characteristic

To compensate for the distortion arising from such a phase characteristic, a network should be designed which introduces little delay at low and high frequencies but considerable delay in the center band of frequencies. Or in terms of the phase characteristic, the compensation network should have an S shaped curve, a large slope in the center band and a small slope outside this band. The desired characteristics of the compensation network are illustrated in Fig. 2, where ω_1 is called the center frequency and is 100 kc for the example of Fig. 1.

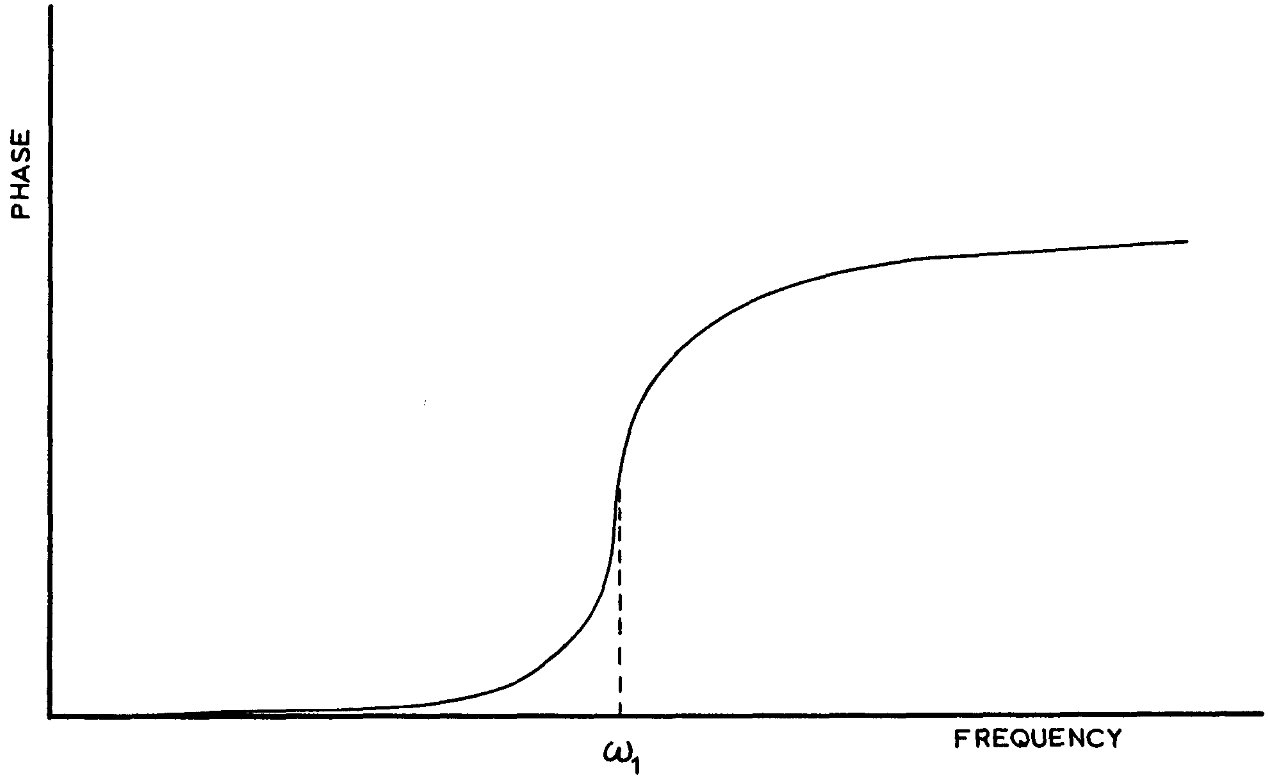


Fig. 2. Desired Characteristic of Compensation Network

Finally in order that the amplitude characteristic be unaffected it is highly desirable that the compensation network be all-pass. We will now consider an example of the design of a network with such characteristics.

Design Procedure

If one restricts their considerations to passive networks, an obvious candidate for the compensation network is the symmetrical lattice network shown in Fig. 3. Network designers have found that this network yields great versatility of specifications.

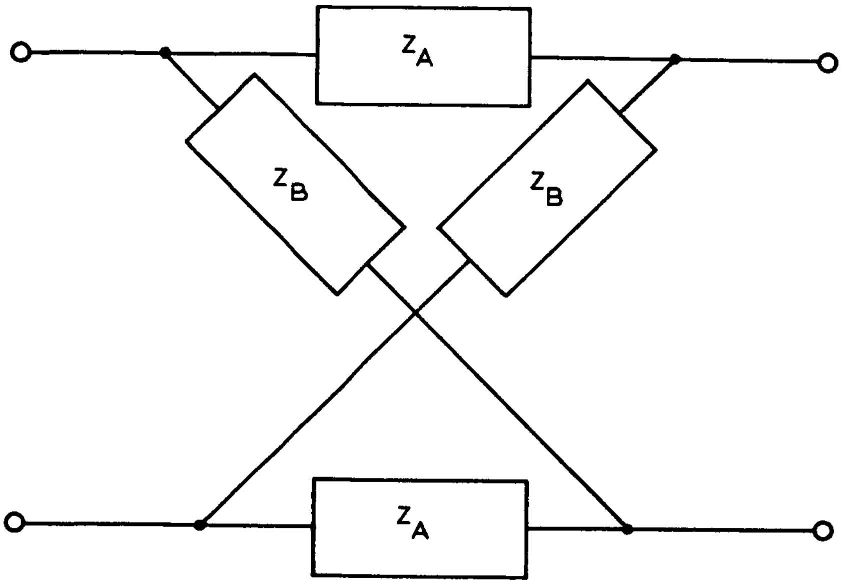


Fig. 3. The Symmetrical Lattice Network

By usual techniques the open and short circuit impedances are found to be

$$Z_{oc} = \frac{1}{2} (Z_A + Z_B) \tag{1}$$

and

$$Z_{sc} = \frac{2 Z_A Z_B}{Z_A + Z_B} \tag{2}$$

from which the characteristic impedance is

$$Z_o = \sqrt{Z_{oc} Z_{sc}} = \sqrt{Z_A Z_B} \tag{3}$$

and the propogation constant is given by

$$\varepsilon^{\gamma} = \varepsilon^{\alpha} \varepsilon^{j\beta} = \frac{1 + \sqrt{Z_A/Z_B}}{1 - \sqrt{Z_A/Z_B}} \tag{4}$$

Now if Z_A and Z_B are pure reactances of opposite sign

$$\varepsilon^{\alpha}\varepsilon^{j\beta} = \frac{1 + j X}{1 - j X} \qquad (X, \text{ real}) \tag{5}$$

this gives

$$\alpha = 0 \tag{6}$$

and the phase is given by

$$\phi = 2 \tan^{-1} X \tag{7}$$

Finally, the characteristic impedance is purely real

$$Z_o = R_o \tag{8}$$

Since $\alpha = 0$, we have an all-pass network with a phase characteristic given by Eq. 7.

The simplest all-pass network (Z_A and Z_B pure reactances of opposite sign) is given by

$$Z_A = j\omega L$$

and

$$Z_B = \frac{1}{j\omega C}$$

From Eq. 7.

$$\phi = 2 \tan^{-1} X = 2 \tan^{-1} \sqrt{LC}\ \omega \tag{9}$$

The slope of this phase characteristic and hence the time delay is a monotonically decreasing function of ω and is therefore not satisfactory. Consider next the symmetrical lattice of Fig. 4.

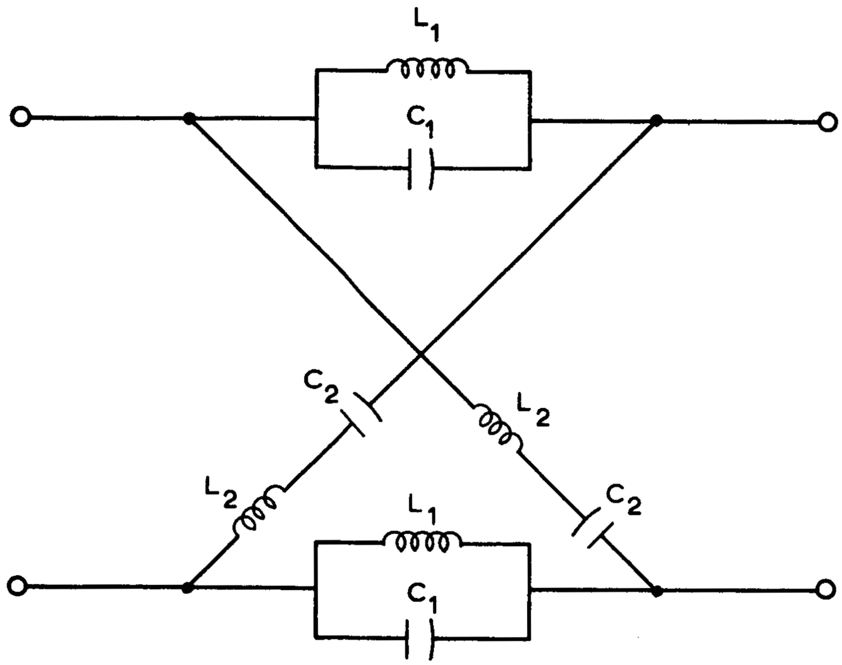


Fig. 4. The Compensation Network

This network has

$$Z_A = \frac{j\omega L_1}{1 - \omega^2 L_1 C_1} \qquad (10)$$

$$Z_B = \frac{1 - \omega^2 L_2 C_2}{j\omega C_2} \qquad (11)$$

and

$$Z_o = \sqrt{\frac{L_1}{C_2} \; \frac{1 - \omega^2 L_2 C_2}{1 - \omega^2 L_1 C_1}} \qquad (12)$$

From Eqs. 10 and 11

$$\frac{Z_A}{Z_B} = \frac{-\omega^2 \; L_1 \; C_2}{(1 - \omega^2 L_1 C_1)(1 - \omega^2 L_2 C_2)} \qquad (13)$$

From Eq. 4, it is apparent that this network will be all-pass if the ratio given by Eq. 13 is negative for all ω. This condition is satisfied if

$$L_1C_1 = L_2C_2 = b \tag{14}$$

For this all-pass condition, the phase is given by Eq. 7 as

$$\phi = 2 \tan^{-1}\left[\frac{\omega\sqrt{L_1C_2}}{1 - \omega^2 L_1C_1}\right]$$

If we let $\sqrt{L_1C_2} = a$, we have

$$\phi = 2 \tan^{-1}\left[\frac{a\,\omega}{1 - b\,\omega^2}\right] \tag{15}$$

If one examines Eq. 15 for ϕ vs. ω characteristics with a and b as parameters, it is readily seen that the desired characteristic of Fig. 2 is obtained. Further the amount of compensation in the center band is determined by the relationship between a and b. In order to establish a measure of the amount of compensation and to properly place the band of maximum compensation, it is necessary to investigate Eq. 15 in detail.

An S shaped curve of the type described by Eq. 15 must have a point of inflection. This point of inflection will be taken as the center frequency ω_1. The point of inflection is given by $\frac{d^2\phi}{d\omega^2} = 0$. From Eq. 15 this gives

$$b^3\,\omega_1^{\,4} + 2b^2\,\omega_1^{\,2} + a^2 - 3b = 0$$

which has the solution

$$\omega_1^2 = \frac{1}{b}\left[-1 \pm \sqrt{1 - \frac{a^2 - 3b}{b}}\right] \qquad (16)$$

This result yields a real and positive center frequency ω_1 if

$$a^2 < 3b \qquad (17)$$

Two cases are of interest

$$1. \quad \omega_1^2 = \frac{1}{b} \quad \text{if } a^2 << b \qquad (18)$$

$$2. \quad \omega_1^2 = \frac{0.732}{b} \quad \text{if } a^2 = b \qquad (19)$$

The amount of compensation in the center band will depend on the degree to which the inequality (17) is satisfied. For $a^2 \leq \frac{1}{10}$ b, $\omega_1^2 \approx \frac{1}{b}$

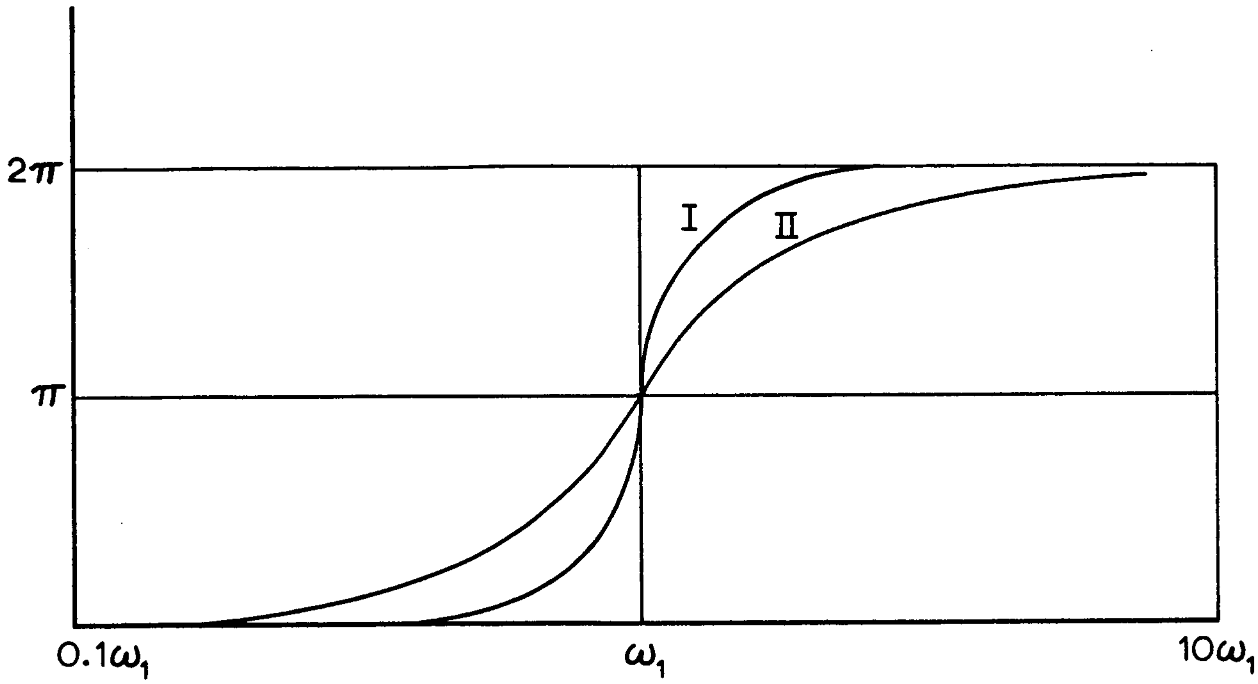
and the design equations become

$$L_1C_1 = L_2C_2 = \frac{1}{\omega_1^2} \qquad (20)$$

$$Z_o = R_o = \sqrt{\frac{L_1}{C_2}} \qquad (21)$$

$$L_1 \; C_2 \leq \frac{1}{10 \quad \omega_1^2} \qquad (22)$$

A pair of phase frequency curves for these results are shown in Fig. 5 for two different relations for inequality (22).



I: $L_1 C_2 = \dfrac{1}{100\omega_1^2}$ II: $L_1 C_2 = \dfrac{1}{10\,\omega_1^2}$

Fig. 5. Phase vs. Frequency for the Compensation Network.

It can be seen that the designer has two degrees of freedom. For example he can choose the characteristic resistance and the center frequency and then select values for the elements to satisfy (22). The phase will then be given by

$$\phi = 2 \tan^{-1} \frac{\omega \sqrt{L_1 C_2}}{1 - \left(\frac{\omega}{\omega_1}\right)^2} \qquad (23)$$

<u>Problem</u>

Let it be required to design a compensation network having the characteristic of Fig. 5, curve II with a characteristic resistance of 100 ohms and a center frequency of 100 k.c. Further suppose the phase variation is desired over 4π radians.

References

1. Van Valkenberg, M. E., Modern Network Synthesis, John Wiley and Sons, New York, 1960

2. Weinberg, Louis, Network Analysis and Synthesis, McGraw Hill, 1962

Problem Solution

Two identical stages in cascade will be used to obtain the desired phase variation. From the design equations

$$L_1 C_2 = \frac{1}{10(2\pi)^2 \times 10^{10}}$$

$$L_1 C_1 = L_2 C_2 = \frac{2}{(2\pi)^2 \times 10^{10}}$$

and

$$\frac{L_1}{C_2} = 10^4$$

From which the element values are

$$L_1 = \frac{1}{\sqrt{10}\,(2\pi)} \quad \text{millihenry}$$

$$L_2 = \frac{\sqrt{10}}{2\pi} \quad \text{millihenry}$$

$$C_1 = \frac{0.2\sqrt{10}}{2\pi} \quad \text{microfarad}$$

$$C_2 = \frac{0.1}{\sqrt{10}\,(2\pi)} \quad \text{microfarad}$$